

CBGS SCHEME



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18MATDIP31

Third Semester B.E. Degree Examination, July/August 2021 Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

- 1 a. Show that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1} \cos^n\left(\frac{\theta}{2}\right) \cos\left(\frac{n\theta}{2}\right)$. (07 Marks)
- b. Express $1 - i\sqrt{3}$ in polar form and hence find its modulus and amplitude. (06 Marks)
- c. Express $\frac{1}{1 - \cos\theta + i\sin\theta}$ in the form $a + ib$ and also find its conjugate. (07 Marks)
- 2 a. Define dot product between two vectors A and B. Find the sine of the angle between the vectors $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - 2\hat{j} + 2\hat{k}$. (07 Marks)
- b. If $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{B} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{C} = 3\hat{i} + \hat{j}$, find the value of p such that $\vec{A} - p\vec{B}$ is perpendicular to \vec{C} . (06 Marks)
- c. Find $\vec{a} \cdot (\vec{b} \times \vec{c})$, $\vec{b} \times (\vec{a} \times \vec{c})$ and $\vec{c} \cdot (\vec{a} \times \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$, $\vec{c} = 3\hat{i} - \hat{j} - \hat{k}$. (07 Marks)
- 3 a. Obtain the Maclaurin's series expansion of $\log(\sec x)$ upto the terms containing x^6 . (07 Marks)
- b. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ then using Euler's theorem, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. (06 Marks)
- c. If $u = f(x - y, y - z, z - x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (07 Marks)
- 4 a. Obtain the Maclaurin's series expansion of the function $\sqrt{1 + \sin 2x}$ upto x^4 . (07 Marks)
- b. If $u = e^{\frac{x^2 y^2}{x+y}}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u \log u$ using Euler's theorem. (06 Marks)
- c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$ (07 Marks)
- 5 a. A particle moves along a curve $x = 3t^2$, $y = t^3 - 4t$, $z = 3t + 4$ where t is the time variable. Determine the components of velocity and acceleration vectors at $t = 2$ in the direction $\hat{i} - 2\hat{j} + 2\hat{k}$. (07 Marks)
- b. Find the unit normal vector to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. (06 Marks)
- c. Show that the vector field $\vec{F} = (2x + yz)\hat{i} + (4y + zx)\hat{j} - (6z - xy)\hat{k}$ is irrotational. Also find ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)



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- 6 a. Find $\text{div } \vec{F}$ and $\text{Curl } \vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (07 Marks)
- b. If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y^4)\hat{j} - 2x^3z^2\hat{k}$ then find $\nabla \cdot \vec{F}$, $\nabla \times \vec{F}$ and $\nabla \cdot (\nabla \times \vec{F})$ at $(2, -1, 0)$. (06 Marks)
- c. Determine the constant 'a' such that the vector $(2x + 3y)\hat{i} + (ay - 3z)\hat{j} + (6x - 12z)\hat{k}$ is Solenoidal. (07 Marks)
- 7 a. Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x dx$ ($n > 0$). (07 Marks)
- b. Evaluate $\int_0^a x^4 \sqrt{a^2 - x^2} dx$. (06 Marks)
- c. Evaluate $\int_1^5 \int_1^{x^2} x(x^2 + y^2) dx dy$. (07 Marks)
- 8 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x dx$ ($n > 0$). (07 Marks)
- b. Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$ (06 Marks)
- c. Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dy dx dz$ (07 Marks)
- 9 a. Solve $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$ (07 Marks)
- b. Solve $\frac{dy}{dx} - y \tan x = y^2 \sec x$. (06 Marks)
- c. Solve $3x(x + y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$ (07 Marks)
- 10 a. Solve $\frac{dy}{dx} + y \cot x = \sin x$. (07 Marks)
- b. Solve $(x + 3y - 4)dx + (3x + 9y - 2)dy = 0$ (06 Marks)
- c. Solve $[1 + (x + y) \tan y] \frac{dy}{dx} + 1 = 0$ (07 Marks)
